Phase Estimation in Spacekime Representation

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Overview

- Goal: treat 'time' as a complex variable and to estimate its phase.

- Intuition: the observed data is a mixture of two underlying processes.

- Strategy:
  1. Identify the hidden processes.
  2. Set them as basis, and to examine the coefficients of the observed data in terms of this basis.

Signal processing

1. Transform the signal into frequency space.
2. Divide the frequencies into two parts (e.g. high frequencies and low ones).
3. Transform two parts back as basis.
4. Estimate the phase between the original signal and the basis signals by

$$\varphi = \arccos \frac{\langle f, f_1 \rangle}{\|f\| \|f_1\|}$$

Signal and its decomposition
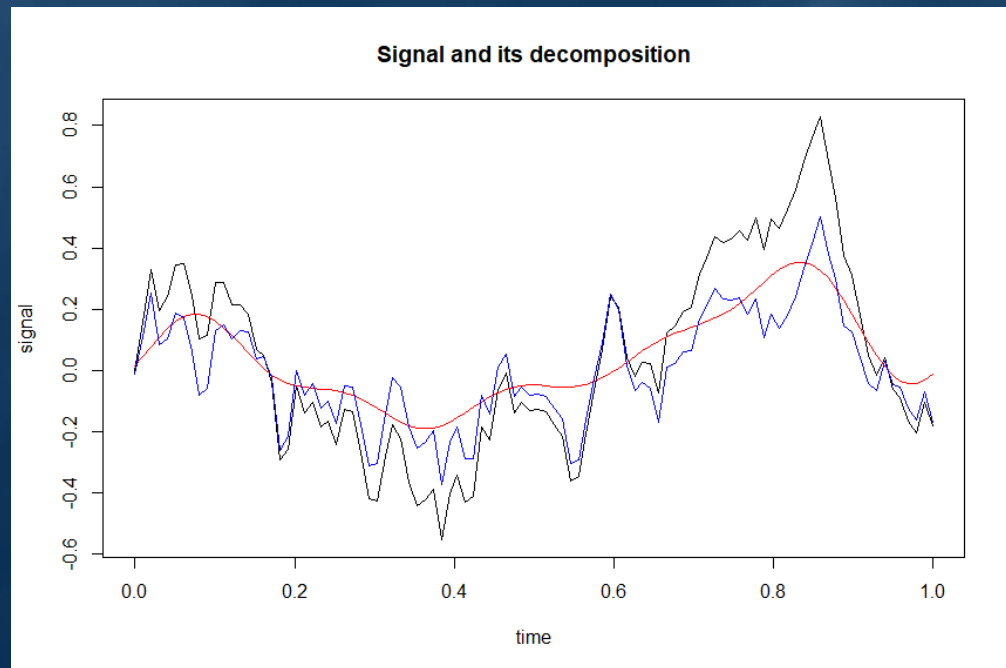
Mixture Probabilistic Model

- Model the underlying processes follow some distributions (random processes).
- The observed process is a probabilistic mixture.

$$X = \pi X_1 + (1 - \pi) X_2$$

- The phase can be estimated by

$$\varphi = \arctan\left(\frac{\pi}{1 - \pi}\right)$$

Application on economics forcasting

Spacekime ARIMAX Analytics (Train: 2000-2014; Test: 2015-2017) GDP (PPS) Forecasting

based on fitting ARIMAX Models on spline interpolated & kime-transformed Belgium data

- ····· Belgium Training Data (2000-2014)
- ——— ARIMAX(4,0,2)-model GDP Forecasting (2015-2017)
- – – – ARIMAX(2,0,1) Belgium Xreg Nil-Phase Reconstruction (2015-2017)
- ——— ARIMAX(1,0,0) Belgium Xreg Swapped-Phase Reconstructions (2015-2017)
- ——— ARIMAX(0,0,0)(2,0,0)[20] Belgium Xreg Random-Phase Reconstructions (2015-2017)
- ——— Belgium Official Reported GDP (2015-2017)

Training Region (2000-2014)

$\text{Model(GDP)} \rightarrow \text{ARIMAX}(p, q, r) \; ; \; \text{XReg} \equiv X_i \; i \in 1:131$

Validation Region (2015-2017)

$\hat{\text{GDP}} \leftarrow \text{ARIMAX}(.,.,.); \; \text{XReg} \equiv X_i \; i \in 1:131$